Pre-class Warm-up!!!

Which of the following statements is logically equivalent to the statement that the square matrix A is invertible?

- a. The equation Ax = 0 is consistent $A \times = 0$ is always ansistent, b. The reduced echelon form of A is the identity \implies we can compute A⁻¹ by Gauss-Jordan \overrightarrow{U} c. The echelon form of A has a leading entry in every column
 - d. None of the above
 - e. More than one of the above.

- IF A is invertible then every equation A = b has a solution : $x = A^{-1}b$
- so the echelon fam has a leading entry
- in every row. Lence a leading entry in every column: C.

3.6 Determinants

Laplace expansion

We learn:

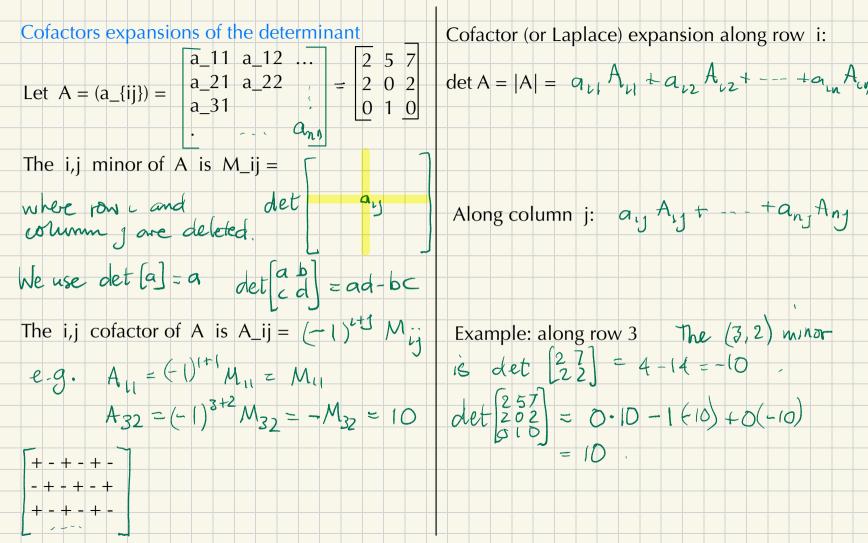
- the definition in terms of cofactor expansions
 row and column properties, the effect under elementary operations
 - computation using Gaussian elimination

New vocabulary:

minors, cofactors, adjoint matrix, transpose matrix, upper triangular matrix

Some theorems:

- the formula for the inverse matrix using the adjoint matrix
- the determinant of the product is the product of the determinants
- Cramer's rule



Row and column properties of the determinant

 cofactor expansions along any row or along any column are equal

The effect of elementary row operations on det:

- adding a multiple of a row to another row leaves the determinant unchanged
- switching two rows multiplies the determinant by -1
- multiplying a row by a number t multiplies the determinant by t.

Page 201 question 10: Evaluate the determinant after simplifying by adding a multiple of some row or column to another.

Solution. The det is unchanged by adding 2. column 1 to column 2

= 28

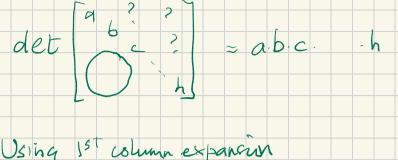
We get (-305) 206 cofactor expansions along any row or along any column are equal

The effect of row operations:

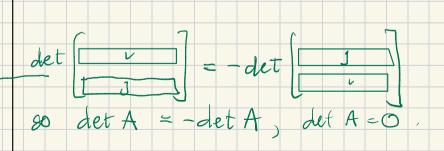
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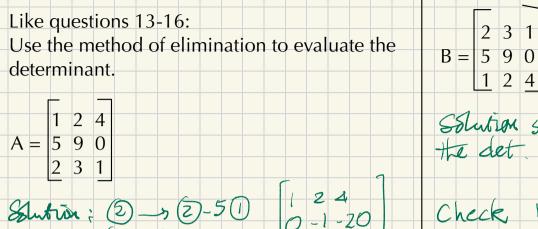
Consequences

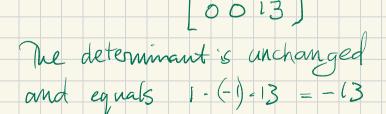
- If a matrix has a zero row or column then
 det = 0 from the Laplace expansion.
- If a matrix has two rows the same or two columns the same then det = 0
- The det of a triangular matrix is the product of the diagonal elements.



$$\det \begin{bmatrix} 3 \\ -3 \end{bmatrix} = a \begin{bmatrix} 5 \\ -2 \\ -3 \end{bmatrix} = a \begin{bmatrix} -2 \\ -3 \\ -3 \end{bmatrix} = a \begin{bmatrix} -2 \\ -3 \\ -3 \end{bmatrix} = a \begin{bmatrix} -2 \\ -3 \\ -3 \end{bmatrix}$$



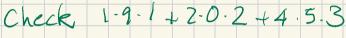


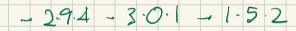


(3) - 2(1)

3

Solution start with Der 3, multiplies the det by -1, so the onswer's 1+13

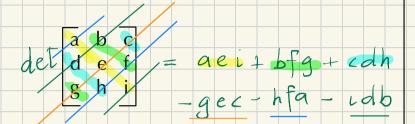




- -13

Practical way to compute the determinant of

a 3 x 3 matrix.



Pre-class Warm-up!!! a. -1

b. 0

c. 1

d. 2

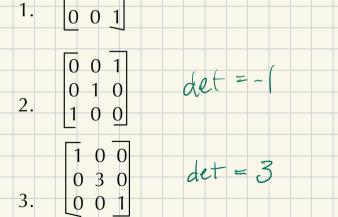
e. 3

Question. What are the determinants of the three elementary matrices

det=1

3

0



More properties of the determinant

If we mattiply a row by r, the det is multiplied sig r.

a,

det

then

B

a.+b. = det A + det B,

Defining properties:

det A is the unique function on square matrices for which

- it is linear on each row ~
- If we switch two rows, det A is multiplied by -1
- det (identity matrix) = 1

It has the same defining properties on columns

- If A, B are n x n matrices then det AB = det A det B
- The transpose matrix: det A^T = det A

$$\begin{array}{c|c} Fxample & \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ \end{bmatrix} & \begin{bmatrix} 1 & 4 & 7 \\ 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \\ \end{bmatrix}$$

This relates to writing invertible matrices as products of elementary matrices.

More properties of the determinant 2 0 3 Question 34 Find A^{-1} when A = -5 -4 2Theorem 5 2 -1 1 If det $A \neq 0$ then A is invertible and it's Solution det A = 35 inverse is A-1 = [Aij 12 = $\begin{bmatrix} M_{ij} \end{bmatrix} = \begin{bmatrix} -2 & -9 & 13 \\ 3 & -4 & -2 \\ 12 & 19 & -8 \end{bmatrix}$ where Aij = (-1) + 1 Mij is the 1,1 cofactor $= \begin{bmatrix} -2 & q & 13 \\ -3 & -4 & 2 \end{bmatrix}$ is called the adjoint matrix. Ail Consequences: A square matrix A is invertible $\langle = \rangle \det A \neq 0$ $\overrightarrow{A} = \frac{1}{35}$ -2 -3 12 -2 -4 -19 13 2 -8F AB = BA = I then det(AB) = det A - det B = det I so det A is invertible. "E" from

Cramer's rule

Question: Solve

